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Generalized Gauge Field Approach
To Lightlike Branes

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Abstract

We propose a general action describing the dynamics of *lightlike (LL) p*-branes in any odd $(p + 1)$ world-volume dimensions. Next, we consider self-consistent coupling of *LL*-membranes ($p{=}2$) to $D{=}4$ Einstein-Maxwell system plus a $D{=}4$ three-index antisymmetric tensor gauge field. The *LL*-brane serves as a material and charge source for gravity and electromagnetism and, furthermore, it produces a dynamical space-varying cosmological constant. We present static spherically-symmetric solutions where the space-time consists of two regions with different black-hole-type geometries and different values for dynamically generated cosmological constant, separated by the *LL*-brane which "straddles" their common event horizon.

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1. Introduction

In the context of non-perturbative string theory there arise several types of higher-dimensional membranes (*p*-branes, *Dp*-branes) which play a crucial role in the description of string dualities, microscopic physics of black holes, gauge theory/gravity correspondence [1], cosmological brane-world scenarios [2], model building in high-energy particle phenomenology [3], *etc*.

There is a distinct class of branes – *lightlike branes*, which are of particular interest in general relativity. They describe impulsive lightlike signals arising in various cataclysmic astrophysical events [4]. Lightlike membranes are basic ingredients in the so called "membrane paradigm" theory [5] of black hole physics. Furthermore, in the context of the so called thin-wall description of domain walls coupled to gravity [6, 7] they are able to provide quite effective treatment of many cosmological and astrophysical effects.

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In refs.[6, 7] lightlike branes in the context of gravity and cosmology have been extensively studied from a phenomenological point of view, *i.e.*, by introducing them without specifying the Lagrangian dynamics from which they may originate. On the other hand, in a series of recent papers [9, 10] we have developed a new field-theoretic approach for a systematic description of the dynamics of lightlike branes starting from concise *Weyl-conformally invariant* actions. The latter are related to, but bear significant qualitative differences from, the standard Nambu-Goto-type *p*-brane actions[1].

The main aim of the present papers is to show that there exists a general class of (not necessarily Weyl-conformally invariant) consistent Lagrangian theories of lightlike branes, which is universal in the sense that all these theories yield physically equivalent solutions of the equations of motion, especially when coupled to bulk gravity-matter systems (see Section 4 below).

Our approach is based on two basic ingredients:

- Employing alternative non-Riemannian integration measure (volume-form) [11, 12] in the actions of generally-covariant (reparametrization-invariant) field theories instead of (or, more generally, on equal footing with) the standard Riemannian volume form.

- Employing auxiliary world-volume gauge field with a Lagrangian being an arbitrary function of the standard Maxwell Lagrangian term.

Before proceeding to the main exposition let us briefly recall the standard Polyakov-type formulation of the ordinary (bosonic) Nambu-Goto *p*-brane action:

$$S = -\frac{T}{2} \int d^{p+1}\sigma \sqrt{-\gamma} \left[\gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \Lambda(p-1) \right] . \quad (1)$$

Here γ_{ab} is the ordinary Riemannian metric on the $p+1$-dimensional brane world-volume with $\gamma \equiv \det \|\gamma_{ab}\|$. The world-volume indices $a, b = 0, 1, \ldots, p$; $G_{\mu\nu}$ denotes the Riemannian metric in the embedding space-time with space-time indices $\mu, \nu = 0, 1, \ldots, D - 1$. T is the given *ad hoc* constant brane tension; the constant Λ can be absorbed by rescaling T. The equations of motion w.r.t. γ^{ab} and X^μ read:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu} + \gamma_{ab} \frac{\Lambda}{2}(p-1) = 0 , \quad (2)$$

$$\partial_a \left(\sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 , \quad (3)$$

where:

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} G^{\mu\kappa} \left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda} \right) \quad (4)$$

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[1]In ref.[8] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

is the Christoffel connection for the external metric. In particular, when $p \neq 1$ Eqs.(2) imply:

$$\Lambda \gamma_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \ . \tag{5}$$

Let us note the following properties of standard Nambu-Goto p-branes manifesting their crucial differences w.r.t. the lightlike branes discussed below. Eq.(5) tells us that: (i) the induced metric on the Nambu-Goto p-brane world-volume is *non-singular*; (ii) standard Nambu-Goto p-branes describe intrinsically *massive* modes.

2.   Lightlike Branes. Action and Equations of Motion

Let us consider the following new kind of p-brane action involving modified world-volume integration measure density $\Phi(\varphi)$ and an auxiliary (Abelian) world-volume gauge field A_a:

$$S = - \int d^{p+1}\sigma \, \Phi(\varphi) \Big[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - L\big(F^2\big) \Big] \tag{6}$$

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \ldots i_{p+1}} \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \varphi^{i_1} \ldots \partial_{a_{p+1}} \varphi^{i_{p+1}} \tag{7}$$

$$F^2 \equiv F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd} \tag{8}$$

Here γ_{ab} denotes the intrinsic Riemannian metric on the brane world-volume, $\gamma = \det \|\gamma_{ab}\|$, $F_{ab} = \partial_a A_b - \partial_b A_a$ and $a, b = 0, 1, \ldots, p; i, j = 1, \ldots, p+1$. $L\big(F^2\big)$ is an arbitrary function of the Maxwell Lagrangian term. As we will see below, consistency of dynamics requires $F^2 L'\big(F^2\big) > 0$ (the prime on L indicating derivative w.r.t. its argument).

Rewriting the action (6) in the following equivalent form:

$$S = - \int d^{p+1}\sigma \, \chi \sqrt{-\gamma} \Big[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - L\big(F^2\big) \Big] \quad , \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \tag{9}$$

we see that the composite field χ plays the role of a dynamical (variable) brane tension. Let us note the following differences of (6) (or (9)) w.r.t. the standard Nambu-Goto p-branes (in the Polyakov-like formulation) (1):

- New non-Riemannian integration measure density $\Phi(\varphi)$ instead of the usual $\sqrt{-\gamma}$, and no "cosmological-constant" term $((p-1)\sqrt{-\gamma})$.

- Variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$.

- Auxiliary world-sheet gauge field A_a entering via arbitrary non-linear Lagrangian.

- Possibility for natural couplings of auxiliary A_a to external world-volume ("color" charge) currents J^a.

- The action (6) describes *intrinsically light-like* p-branes for any even p, *i.e.*, any odd-dimensional world-volume, see below.

- For the special choice $L\left(F^2\right) = \sqrt{F^2}$ the brane action (6) becomes Weyl-conformally invariant for *any* p [9, 10]. Also, let us note that there are *no quantum conformal anomalies* in odd $(p+1)$ dimensions!

Employing the short-hand notations (8) and:

$$(\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \qquad (10)$$

for the induced metric, the equations of motion w.r.t. measure-building auxiliary scalars φ^i and γ^{ab} read, respectively:

$$\frac{1}{2}\gamma^{cd}\left(\partial_c X \partial_d X\right) - L\left(F^2\right) = M \ \left(= \text{integration const}\right), \qquad (11)$$

$$\frac{1}{2}\left(\partial_a X \partial_b X\right) + 2L'\left(F^2\right)F_{ac}\gamma^{cd}F_{db} = 0, \qquad (12)$$

where the latter can be viewed as p-brane analogues of the string Virasoro constraints.

Eqs.(11)–(12) have the following profound consequences. First, from (12) we obtain:

$$\det\|\left(\partial_a X \partial_b X\right)\| = \left(-4L'(F^2)\right)^{p+1}\left(-\det\|\gamma_{ab}\|\right)\left(\det\|iF_{ab}\|\right)^2 \qquad (13)$$

For $(p+1) =$even world-volume dimensions the r.h.s. of (13) is strictly positive (because of the Lorentzian signature of γ_{ab}), whereas the determinant of the induced metric in the l.h.s of (13) should be negative conforming with the Lorentzian signatures of both world-volume and embedding space-time metrics. Therefore, we conclude that the brane action (6) does not describe a consistent dynamics for even world-volume dimensions.

Next, taking the trace in (12) and comparing with (11) implies the following crucial relation for the Lagrangian function $L\left(F^2\right)$:

$$L\left(F^2\right) - 2F^2 L'\left(F^2\right) + M = 0. \qquad (14)$$

Eq.(14) can be viewed in two ways:

(a) For $M = 0$ (14) is identically satisfied if we choose $L\left(F^2\right) = \sqrt{F^2}$. This is precisely the case of *Weyl-conformally invariant* brane action (6) which was introduced and extensively studied in Refs.[9, 10].

(b) For arbitrary (non-zero) constant M and arbitrary function $L\left(F^2\right)$ Eq. (14) determines F^2 as certain function of M, *i.e.*

$$F^2 = F^2(M) = \text{const} \qquad (15)$$

This is the generic case which will be discussed in what follows.

The third and most important implication of Eqs.(12) is as follows. Since F_{ab} is anti-symmetric $(p+1)\times(p+1)$ matrix, then F_{ab} is *not invertible* in any odd $(p + 1)$ – it has at least one zero-eigenvalue vector-field V^a:

$F_{ab}V^b = 0$. Therefore, for any odd $(p+1)$ the induced metric $(\partial_a X \partial_b X)$ on the world-volume of the p-brane model (6) is *singular* (as opposed to the ordinary Nambu-Goto brane, see (5)):

$$(\partial_a X \partial_b X) V^b = 0 \quad , \quad \text{i.e.} \quad (\partial_V X \partial_V X) = 0 \ , \ (\partial_\perp X \partial_V X) = 0 \qquad (16)$$

where $\partial_V \equiv V^a \partial_a$ and $\partial_\perp$ are derivatives along the tangent vectors in the complement of V^a. In particular, for $(p+1) = 3$ we have $V^a \simeq \frac{1}{2} \frac{\varepsilon^{abc}}{\sqrt{-\gamma}} F_{bc}$.

Thus, we arrive at the following important conclusion: every point on the world-surface of the p-brane (6) (for odd $(p+1)$) moves with the speed of light in a time-evolution along the zero-eigenvalue vector-field V^a of F_{ab}. Therefore, we will name (6) (for odd $(p+1)$) by the acronym *LL-brane* (Lightlike-brane) model.

Finally, we get the equations of motion w.r.t. auxiliary gauge field A_a:

$$\partial_b \left(F_{cd} \gamma^{ac} \gamma^{bd} \sqrt{-\gamma} \, \chi \right) = 0 \ , \qquad (17)$$

where relation (15) has been taken into account, and the equations of motion w.r.t. X^μ :

$$\partial_a \left(\chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \ . \qquad (18)$$

Remark. In what follows we will use a natural ansatz for the world-volume electric field $F_{0i} = 0$ implying that $(V^a) = (1, \underline{0})$, i.e., $\partial_V = \partial_0 \equiv \partial_\tau$.

3. Special Case $p = 2$. Coupling to Space-Time Maxwell and Rank-3 Antisymmetric Tensor Gauge Fields

Henceforth we will explicitly consider the special case $p = 2$ of (6), i.e., the lightlike membrane model:

$$S = - \int d^3\sigma \, \Phi(\varphi) \left[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - L(F^2) \right] \qquad (19)$$

$$\Phi(\varphi) \equiv \frac{1}{3!} \varepsilon_{ijk} \varepsilon^{abc} \partial_a \varphi^i \partial_b \varphi^j \partial_c \varphi^k \quad , \quad a, b, c = 0, 1, 2 \ , \ i, j, k = 1, 2, 3 \ . \quad (20)$$

Invariance under world-volume reparametrizations allows to introduce the standard (synchronous) gauge-fixing conditions:

$$\gamma^{0i} = 0 \ (i = 1, 2) \quad , \quad \gamma^{00} = -1 \ . \qquad (21)$$

The ansatz $F_{0i} = 0$, together with the Biancchi identity $\varepsilon^{abc} \partial_a F_{bc} = 0$ and (21) when inserted in (15), implies:

$$F^2 = 2B^2 = \text{const} \quad ; \quad B = \frac{1}{2} \frac{\varepsilon^{ij}}{\sqrt{\gamma^{(2)}}} F_{ij} \quad , \quad \gamma^{(2)} \equiv \det \|\gamma_{ij}\| \ ; \qquad (22)$$

$$\partial_0 \left(\varepsilon^{ij} F_{ij} \right) = 0 \ \longrightarrow \ \partial_0 \gamma^{(2)} = 0 \ . \qquad (23)$$

Then the gauge-fixed equations motion for A_a (17) drastically simplify:

$$\partial_i \chi = 0 \ , \tag{24}$$

where $\chi \equiv \frac{\Phi(\phi)}{\sqrt{-\gamma}}$ (the dynamical brane tension as in (9)).

Employing (21), the remaining gauge-fixed equations of motion w.r.t. γ^{ab} and X^μ read (recall $(\partial_a X \partial_b X) \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$):

$$(\partial_0 X \partial_0 X) = 0 \quad , \quad (\partial_0 X \partial_i X) = 0 \ , \tag{25}$$

$$(\partial_i X \partial_j X) - 2c_1(M)\gamma_{ij} = 0 \quad , \quad c_1(M) \equiv F^2 L'(F^2)\Big|_{F^2 = F^2(M)} = \text{const} \, , \tag{26}$$

$$\Box^{(3)} X^\mu + \chi \sqrt{\gamma^{(2)}} \left(-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda \right) \Gamma^\mu_{\nu\lambda} = 0 \ , \tag{27}$$

$$\Box^{(3)} \equiv -\partial_0 \left(\chi \sqrt{\gamma^{(2)}} \partial_0 \right) + \partial_i \left(\chi \sqrt{\gamma^{(2)}} \gamma^{ij} \partial_j \right) \tag{28}$$

Remark. Note that (26) coincides with the *space-like* part of the Nambu-Goto brane constraint Eqs.(5), whereas (25) drastically differ from their Nambu-Goto counterparts.

Remark. Consistency according to (26) requires that the constant:

$$c_1(M) \equiv F^2 L'(F^2)\Big|_{F^2 = F^2(M)} > 0 \ , \tag{29}$$

therefore, an admissible choice for $L(F^2)$ is:

$$L(F^2) = +\frac{1}{4} F^2 \quad , \quad i.e. \ F^2 = 4M \ , \ c_1(M) = M \tag{30}$$

by virtue of Eq.(14). The term (30) is a Maxwell Lagrangian with a *wrong* sign, however, this is not a contradiction since in the context of *LL*-brane (6) the world-volume gauge field A_a is an auxiliary non-dynamical field.

Remark. In the special case of Weyl-conformally invariant lightlike branes [9, 10], *i.e.*, for $L(F^2) = \sqrt{F^2}$ and $M = 0$ Eqs.(23) and (26) change accordingly to $\partial_0 \left(B \sqrt{\gamma^{(2)}} \right) = 0$ and $(\partial_i X \partial_j X) - \sqrt{2} B \gamma_{ij} = 0$ (the magnetic field B is not necessarily constant in this case).

We can extend straightforwardly the *LL*-brane model (19) via couplings to external space-time electromagnetic field $\mathcal{A}_\mu$ and, furthermore, to external space-time rank 3 gauge potential $\mathcal{A}_{\mu\nu\lambda}$ (Kalb-Ramond-type coupling):

$$S = -\int d^3\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - L\big(F^2\big)\Big]$$

$$-q\int d^3\sigma\, \varepsilon^{abc}\mathcal{A}_\mu \partial_a X^\mu F_{bc} \;-\; \frac{\beta}{3!}\int d^3\sigma\, \varepsilon^{abc}\partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \mathcal{A}_{\mu\nu\lambda} \quad (31)$$

The second Chern-Simons-like term in (31) is a special case of a class of Chern-Simons-like couplings of extended objects to external electromagnetic fields proposed in ref.[13].

Let us recall the physical significance of $\mathcal{A}_{\mu\nu\lambda}$ [14]. In $D = 4$ when adding kinetic term for $\mathcal{A}_{\mu\nu\lambda}$ coupled to gravity (see Eq.(37) below), its field-strength:

$$\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]} = \mathcal{F}\sqrt{-G}\varepsilon_{\kappa\lambda\mu\nu} \quad (32)$$

with a single independent component $\mathcal{F}$ produces *dynamical (positive) cosmological constant*:

$$K = \frac{4}{3}\pi G_N \mathcal{F}^2 \qquad \big(G_N - \text{Newton constant}\big). \quad (33)$$

The constraints (25)–(26) (the gauged-fixed equations of motion w.r.t. γ^{ab}) remain unaltered for the action (31). Using the same gauge choice $\big(\gamma^{0i} = 0, \gamma^{00} = -1\big)$ and ansatz for the world-volume gauge field-strength $\big(F_{0i}(A) = 0\big)$, the equations of motion w.r.t. A_a now acquire the form (recall $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ – the brane tension, $\mathcal{F}_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$):

$$\partial_i X^\mu \partial_j X^\nu \mathcal{F}_{\mu\nu}(\mathcal{A}) = 0 \quad , \quad \partial_i \chi + \frac{\sqrt{2}q}{c_2(M)}\partial_0 X^\mu \partial_i X^\nu \mathcal{F}_{\mu\nu}(\mathcal{A}) = 0 \,, \quad (34)$$

where:

$$c_2(M) \equiv 2\sqrt{F^2}L'(F^2)\Big|_{F^2=F^2(M)} = \text{const}. \quad (35)$$

In particular $c_2(M) = \sqrt{M}$ for the wrong-sign Maxwell choice (30).

Eqs.(34) tell us that consistency of charged *LL*-brane dynamics implies that the external space-time Maxwell field must have zero magnetic component normal to the brane, as well as that the projection of the external electric field along the brane must be proportional to the gradient of the brane tension. Finally, the X^μ equations of motion for (31) read:

$$\Box^{(3)}X^\mu + \chi\sqrt{\gamma^{(2)}}\left(-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl}\partial_k X^\nu \partial_l X^\lambda\right)\Gamma^\mu_{\nu\lambda}$$

$$-2qB\sqrt{\gamma^{(2)}}\partial_0 X^\nu \mathcal{F}_{\lambda\nu}\, G^{\lambda\mu} - \frac{\beta}{3!}\varepsilon^{abc}\partial_a X^\kappa \partial_b X^\lambda \partial_c X^\nu G^{\mu\rho}\, \mathcal{F}_{\rho\kappa\lambda\nu} = 0 \,, \quad (36)$$

where $\mathcal{F}_{\rho\kappa\lambda\nu}$ is given as in (32) and $B = \sqrt{\frac{1}{2}F^2} = \text{const}$ ($B = \sqrt{2M}$ for the wrong-sign Maxwell choice, recall (22), (30)).

4. Bulk Einstein-Maxwell System Coupled to Lightlike Brane

Now let us consider the coupled Einstein-Maxwell-LL-brane system adding also a coupling to a rank 3 gauge potential:

$$S = \int d^4x \sqrt{-G} \left[\frac{R(G)}{16\pi G_N} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4!2}\mathcal{F}_{\kappa\lambda\mu\nu}\mathcal{F}^{\kappa\lambda\mu\nu} \right] + S_{\text{LL-brane}} . (37)$$

Here $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$, $\mathcal{F}_{\kappa\lambda\mu\nu} = 4\partial_{[\kappa}\mathcal{A}_{\lambda\mu\nu]} = \mathcal{F}\sqrt{-G}\varepsilon_{\kappa\lambda\mu\nu}$ as above, and the LL-brane action is the same as in (31).

The equations of motion for the LL-brane subsystem are the same as (25)–(26), (34)–(36), whereas the Einstein, Maxwell and 3-index gauge field equations read:

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi G_N \left(T^{(EM)}_{\mu\nu} + T^{(rank-3)}_{\mu\nu} + T^{(brane)}_{\mu\nu} \right) , \quad (38)$$

$$\partial_\nu \left(\sqrt{-G}G^{\mu\kappa}G^{\nu\lambda}\mathcal{F}_{\kappa\lambda} \right) + q \int d^3\sigma \, \delta^{(4)}\Big(x - X(\sigma) \Big)\varepsilon^{abc}F_{bc}\partial_a X^\mu = 0 , \quad (39)$$

$$\varepsilon^{\lambda\mu\nu\kappa}\partial_\kappa\mathcal{F} + \beta \int d^3\sigma \, \delta^{(4)}(x - X(\sigma))\varepsilon^{abc}\partial_a X^\lambda \partial_a X^\mu \partial_a X^\nu = 0 . \quad (40)$$

where in the last equation we have used relation (32). The energy-momentum tensors read:

$$T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda}G^{\kappa\lambda} - G_{\mu\nu}\frac{1}{4}\mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda}G^{\rho\sigma}G^{\kappa\lambda} , \quad (41)$$

$$T^{(rank-3)}_{\mu\nu} = \frac{1}{3!}\left[\mathcal{F}_{\mu\kappa\lambda\rho}\mathcal{F}_\nu^{\ \kappa\lambda\rho} - \frac{1}{8}G_{\mu\nu}\mathcal{F}_{\kappa\lambda\rho\sigma}\mathcal{F}^{\kappa\lambda\rho\sigma} \right] = -\frac{1}{2}\mathcal{F}^2 G_{\mu\nu} , \quad (42)$$

$$T^{(brane)}_{\mu\nu} = -G_{\mu\kappa}G_{\nu\lambda}\int d^3\sigma \frac{\delta^{(4)}\Big(x - X(\sigma) \Big)}{\sqrt{-G}} \chi \sqrt{-\gamma}\gamma^{ab}\partial_a X^\kappa \partial_b X^\lambda . \quad (43)$$

(recall $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ – the variable brane tension (9)).

We will be looking for static spherically symmetric solutions of the equations of motion (38)–(40) of the bulk gravity-matter system coupled to a charged LL-brane (37) (we will assume spherical topology for the LL-brane surface). Notice that:

(a) The LL-brane serves as material and charge source for the bulk gravity and electromagnetism;

(b) The resulting space-time will consist of two regions (one "interior" and one "exterior") separated by the hypersurface of the LL-brane world-volume which will impose non-trivial *matching conditions* across itself for

the parameters of the (static) spherically symmetric geometries in both space-time regions.

(c) Consistency of dynamics of the LL-brane – it should yield the same solutions for the LL-brane equations of motion both from the point of view of the interior as well as the exterior bulk gravity-matter.

The general form of spherically-symmetric gravitational background in $D = 4$ reads:

$$(ds)^2 = -A(r,t)(dt)^2 + B(r,t)(dr)^2 + C(r,t)[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2] \quad (44)$$

Concerning point (c) above consider the following ansatz:

$$X^0 \equiv t = \tau \;\; , \;\; X^1 \equiv r = r(\tau, \sigma^1, \sigma^2) \;\; , \;\; X^2 \equiv \theta = \sigma^1 \;\; , \;\; X^3 \equiv \phi = \sigma^2 \quad (45)$$

$$\gamma_{ij} = a(\tau, \sigma^1, \sigma^2) \left((d\sigma^1)^2 + \sin^2(\sigma^1)(d\sigma^2)^2) \right) \quad (46)$$

and substitute it into the LL-brane equations of motion. We get:

- Equations for $r(\tau, \sigma^1, \sigma^2)$ from the lightlike constraints (25):

$$\frac{\partial r}{\partial \tau} = \pm \sqrt{\frac{A}{B}} \;\; , \;\; \frac{\partial r}{\partial \sigma^i} = 0 \quad (47)$$

- A strong restriction on the gravitational background itself due to the Virasoro-like constraints (26). Namely, because of (23) the conformal factor a in (46) must be τ-independent. From this and (26) we deduce:

$$\partial_0 \left(\partial_i X \partial_j X \right) = 0 \;\; \longrightarrow \;\; \frac{dC}{d\tau} \equiv \left(\frac{\partial C}{\partial t} \pm \sqrt{\frac{A}{B}} \frac{\partial C}{\partial r} \right) \Big|_{t=\tau, \, r=r(\tau)} = 0 \quad (48)$$

Eq.(48) tells us that the (squared) sphere radius $R^2 \equiv C(r,t)$ must remain constant along the LL-brane trajectory. For static backgrounds $R^2 \equiv C(r)$ Eqs.(48),(47) imply:

$$r(\tau) = r_0 \;\; (= \text{const}) \;\; , \;\; A(r_0) = 0 \quad (49)$$

Eq.(49) is of primary importance as it shows that *the LL-brane automatically positions itself on the event horizon.*

- The Virasoro-like constraints (26), taking into account (49) and (46), imply:

$$a = \frac{C(r_0)}{2c_1(M)} = \text{const} \quad (50)$$

- LL-brane equations of motion (36) for $X^0 \equiv t$ and $X^1 \equiv r$ turn out to be proportional to each other and reduce to an equation for the variable brane tension χ (9):

$$a\partial_\tau \chi + \chi \, a \, \frac{\frac{\partial}{\partial t}\sqrt{AB} \pm \partial_r A}{\sqrt{AB}} \pm \chi \, \frac{\partial_r C}{\sqrt{AB}}$$

$$\mp \frac{\sqrt{2}q}{c_2(M)} \frac{C}{\sqrt{AB}} \mathcal{F}_{0r} \pm \beta \mathcal{F} C \sqrt{AB} = 0 \quad (51)$$

where $\mathcal{F}$ is the independent component of the rank 4 field-strength (32) and the constant $c_2(M)$ is the same as in (35) (here again one sets at the end $t = \tau$, $r = r(\tau)$).

Thus, following the same procedure as in the case of Weyl-conformally invariant lightlike branes [10] we arrive at the following static spherically symmetric solutions for the system (37). The bulk space-time consists of two regions separated by the LL-brane sitting on ("straddling") a common horizon of the former:

$$(ds)^2 = -A_{(\mp)}(r)(dt)^2 + \frac{1}{A_{(\mp)}(r)}(dr)^2 + r^2[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2]\,, \quad (52)$$

where the subscript $(-)$ refers to the region inside, whereas the subscript $(+)$ refers to the region outside the horizon at $r = r_0 \equiv r_{\text{horizon}}$ (49). The interior region is a Schwarzschild-de-Sitter space-time:

$$A(r) \equiv A_{(-)}(r) = 1 - K_{(-)}r^2 - \frac{2G_N m_{(-)}}{r}\,, \quad \text{for } r < r_0\,, \quad (53)$$

whereas the exterior region is Reissner-Nordström-de-Sitter space-time:

$$A(r) \equiv A_{(+)}(r) = 1 - K_{(+)}r^2 - \frac{2G_N m_{(+)}}{r} + \frac{G_N Q^2}{r^2}\,, \quad \text{for } r > r_0\,, \quad (54)$$

with Reissner-Norström (squared) charge given by:

$$Q^2 = 8\pi\bar{q}^2 r_0^4 \quad, \quad \bar{q} \equiv \frac{q}{c_2(M)}\,. \quad (55)$$

The rank 3 tensor gauge potential together with its Kalb-Rammond-type coupling to the LL-brane produce via Eq.(40) a dynamical space-varying cosmological constant which is different inside and outside the horizon:

$$K_{(\pm)} = \frac{4}{3}\pi G_N \mathcal{F}_{(\pm)}^2 \text{ for } r \geq r_0 \ (r \leq r_0) \quad, \quad \mathcal{F}_{(+)} = \mathcal{F}_{(-)} - \beta\,. \quad (56)$$

The Einstein Eqs.(38) and the X^μ-brane Eqs.(36) yield two matching conditions for the normal derivatives w.r.t. the horizon of the space-time metric components:

$$\big(\partial_r A_{(+)} - \partial_r A_{(-)}\big)\big|_{r=r_0} = -16\pi G_N \chi\,,$$
$$\big(\partial_r A_{(+)} - \partial_r A_{(-)}\big)\big|_{r=r_0} = -\frac{r_0(2\bar{q}^2 + \beta^2)\partial_r A_{(-)}\,|_{r=r_0}}{2\chi + \beta r_0 \mathcal{F}_{(-)}}\,. \quad (57)$$

with $\bar{q}$ as in (55). The matching conditions (57) plus relation (50) allow all physical parameters of the solution, *i.e.*, two spherically symmetric black hole space-time regions "soldered" along a common horizon materialized

by the *LL*-brane, as well as the value of the integration constant M, to be expressed in terms of 3 free parameters $(q, \beta, \mathcal{F})$ where (cf. Eq.(31)):

(a) q is the *LL*-brane surface electric charge density;

(b) β is the *LL*-brane (Kalb-Rammond-type) charge w.r.t. rank 3 space-time gauge potential $\mathcal{A}_{\lambda\mu\nu}$;

(c) $\mathcal{F}_{(-)}$ is the vacuum expectation value of the 4-index field-strength $\mathcal{F}_{\kappa\lambda\mu\nu}$ in the interior region.

For the common horizon radius r_0, the conformal factor a of the internal brane metric (46), the Schwarzschild and Reissner-Nordström masses $m_{(\mp)}$ we obtain ($\bar{q}$ is the same as in (55)):

$$r_0^2 = \left[4\pi G_N \left(\mathcal{F}_{(-)}^2 - \beta\mathcal{F}_{(-)} + \bar{q}^2 + \frac{\beta^2}{2} \right) \right]^{-1}, \tag{58}$$

$$a = \left(\bar{q}^2 + \frac{\beta^2}{2} \right) \left[2\pi G_N \left(-\beta\mathcal{F}_{(-)} + \bar{q}^2 + \frac{\beta^2}{2} \right)^2 \right]^{-1}, \tag{59}$$

$$m_{(-)} = \frac{r_0 \left(\frac{2}{3}\mathcal{F}_{(-)}^2 - \beta\mathcal{F}_{(-)} + \bar{q}^2 + \frac{\beta^2}{2} \right)}{2G_N \left(\mathcal{F}_{(-)}^2 - \beta\mathcal{F}_{(-)} + \bar{q}^2 + \frac{\beta^2}{2} \right)}, \tag{60}$$

$$m_{(+)} = m_{(-)} + \frac{r_0 \left(2\bar{q}^2 + \frac{2}{3}\beta\mathcal{F}_{(-)} - \frac{1}{3}\beta^2 \right)}{2G_N \left(\mathcal{F}_{(-)}^2 - \beta\mathcal{F}_{(-)} + \bar{q}^2 + \frac{\beta^2}{2} \right)} \tag{61}$$

For the brane tension we get accordingly:

$$\chi = \frac{r_0}{2} \left(\bar{q}^2 + \frac{\beta^2}{2} - 2\beta\mathcal{F}_{(-)} \right) \tag{62}$$

Inserting (58)–(59) into relation (50) fixes the value ot the integration constant M. In particular, for the wrong-sign Maxwell choice (30) and $\mathcal{F}_{(-)} = 0$ we get $M = 1/4$.

Using expressions (58)–(61) we find for the slopes of the metric coefficients $A_{(\pm)}(r)$ at $r = r_0$:

$$\partial_r A_{(+)}|_{r=r_0} = -\partial_r A_{(-)}|_{r=r_0} \quad , \quad \partial_r A_{(-)}|_{r=r_0} = 8\pi G_N \chi \tag{63}$$

with χ as in (62). The typical form of $A(r)$ is depicted in Fig.1 below. As shown in refs.[10], this form of $A(r)$ creates a potential "well" in the vicinity of the *LL*-brane lying on the common horizon which acts as a trap for test particles falling toward the horizon.
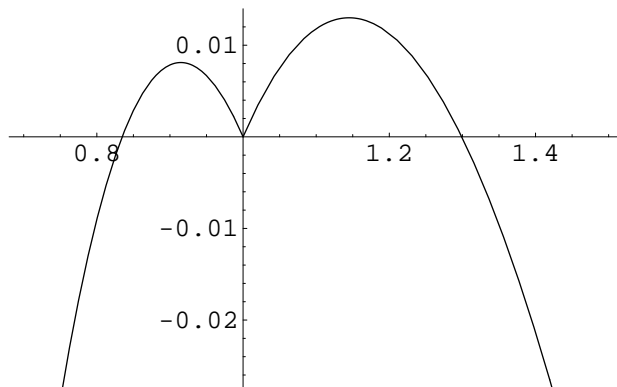
Figure 1: Shape of $A(r)$ as a function of the dimensionless ratio $x \equiv r/r_0$

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